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INVESTIGATION OF HEAT TRANSFER BY CONDUCTION

INTRODUCTION

These Notes describe the simulation, on a PACE® TR-20 General Purpose Analog Computer, of a system involving heat transfer by conduction. It is a system where the temperature-versus-time-and-thickness relationship is required.

Specifically, heat energy in the form of an open flame is stored in a brass block which acts as a heat sink. This heat energy is transferred by conduction to and through a metal rod connected to it. The rod itself is insulated from ambient air and heat losses from the rod to such possible cooling effects during heating are negligible. In this system, shown schematically in Figure 1, it was assumed that heat transfer in the rod is limited to one direction only, that of rod length.

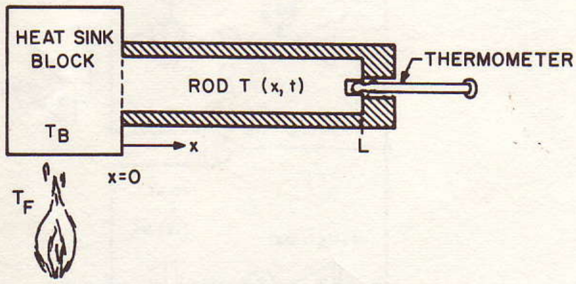

Figure 1: Simplified Diagram of Heat Transfer System

The successful analog computer solution of many engineering problems depends upon the ability of the computer to simulate such systems whose behavior is described by partial differential equations.

SYSTEM EQUATIONS

The basic equations describing this system, based on a fundamental energy balance, are

$$\frac{dT_B}{dt} = \frac{1}{\tau_B}\left[T_F - T_B\right] \quad (1)$$

for <u>heat transfer in the sink block</u>

and

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad (2)$$

for <u>heat transfer in the metal rod</u>

The initial and boundary conditions to be satisfied are

$$\left.\begin{aligned} T_B(o) &= T_A \\ T(o,x) &= T_A \end{aligned}\right\} \text{ambient temperature}$$

$$T(t,o) = T_B = \text{sink temperature}$$

$$\left(\frac{\partial T}{\partial x}\right)_{x=L} = O = \text{insulated rod end}$$

The following variable definitions were used to obtain the final, normalized system equations (listed below) which were used in the simulation:

$$U_B = \frac{T_B - T_A}{T_F - T_A}$$

$$U = \frac{T - T_A}{T_F - T_A}$$

$$Z = \frac{x}{L}$$

$$\theta = \frac{\alpha T}{L^2}$$

Heat Transfer in Sink Block

$$\frac{dU_B}{d\theta} = \left(\frac{L^2}{\alpha \tau_B}\right)\left\{1 - U_B\right\} \quad (3)$$

$$U_B(o) = O$$

Diffusion Equation for Heat Conduction in Rod

$$\frac{\partial U}{\partial \theta} = \frac{\partial^2 U}{\partial z^2} \quad (4)$$

The boundary and initial conditions to be satisfied are

$U(z,o)$	$= O$	Initial Temperature
$U(\theta,o)$	$= U_B$	Surface Temperature
$\left(\frac{\partial U}{\partial z}\right)_{z=1}$	$= O$	Due to Rod Insulation

METHOD OF SOLUTION

Using a second-order central difference approximation for $\partial^2U/\partial z^2$, viz:

$$\frac{\partial^2 U(z,t)}{\partial z^2} = \frac{U(z+\Delta z,t) - 2U(z,t) + U(z-\Delta z,t)}{(\Delta z)^2} \quad (5)$$

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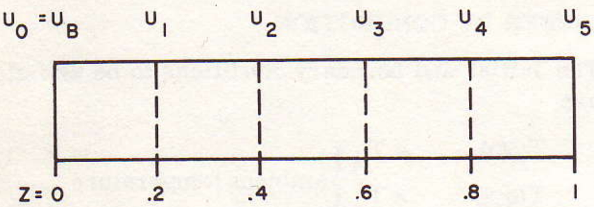
the temperature distribution in the rod is obtained as a function of time for α/L^2 equal to 3/2, 1, and 1/2 seconds $^{-1}$.

The rod is divided into five equally-spaced segments, as shown in Figure 2, in order to derive the scaled equations.



$\Delta Z = 0.2$

$$\left(\frac{1}{\Delta Z}\right)^2 = \frac{1}{0.04} = 25$$

Figure 2: Division of Bar into Equally Spaced Segments

Scaled Equations: Using equation (3), the following scaled equations were obtained:

$$\frac{d\,[10\,U_B]}{d\tau} = \left(\frac{L^2}{\alpha\tau_B\beta}\right)\left[10\,(1-U_B)\right] \qquad (6)$$

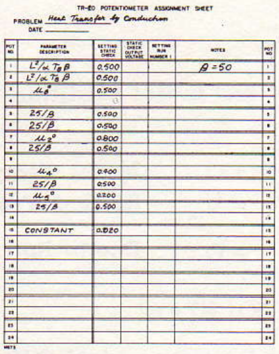
$$\frac{d\,[10\,U_1]}{d\tau} = \left(\frac{25}{\beta}\right)\left[10\,(U_B-U_1) - 10\,(U_1-U_2)\right] \qquad (7)$$

$$\frac{d\,[10\,U_2]}{d\tau} = \left(\frac{25}{\beta}\right)\left[10\,(U_3-U_2) - 10\,(U_2-Y_1)\right] \qquad (8)$$

$$\frac{d\,[10\,U_3]}{d\tau} = \left(\frac{25}{\beta}\right)\left[10\,(U_4-U_3) - 10\,(U_3-U_2)\right] \qquad (9)$$

$$\frac{d\,[10\,U_4]}{d\tau} = \left(\frac{25}{\beta}\right)\left[10\,(U_5-U_4) - 10\,(U_4-U_3)\right] \qquad (10)$$

$$\frac{d\,[10\,U_5]}{d\tau} = \left(\frac{50}{\beta}\right)\left[10\,(U_5-U_4)\right] \qquad (11)$$



TR-20 POTENTIOMETER ASSIGNMENT SHEET

PROBLEM Heat Transfer by Conduction

DATE

POT NO.	PARAMETER DESCRIPTION	SETTING STATIC CHECK	STATIC CHECK OUTPUT VOLTAGE	SETTING RUN NUMBER 1	NOTES	POT NO.
1	$L^2/\alpha\tau_B\beta$	0.500			$\beta = 50$	1
2	$L^2/\alpha\tau_B\beta$	0.500				2
3	u_B^o	0.500				3
4						4
5	$25/\beta$	0.500				5
6	$25/\beta$	0.500				6
7	u_2^o	0.800				7
8	$25/\beta$	0.500				8
9						9
10	u_4^o	0.400				10
11	$25/\beta$	0.500				11
12	u_5^o	0.200				12
13	$25/\beta$	0.500				13
14						14
15	CONSTANT	0.020				15
16						16
17						17
18						18
19						19
20						20
21						21
22						22
23						23
24						24

Figure 4: Potentiometer Assignment Sheet

COMPUTER DIAGRAM

Figure 3 shows the computer diagram for this simulation.

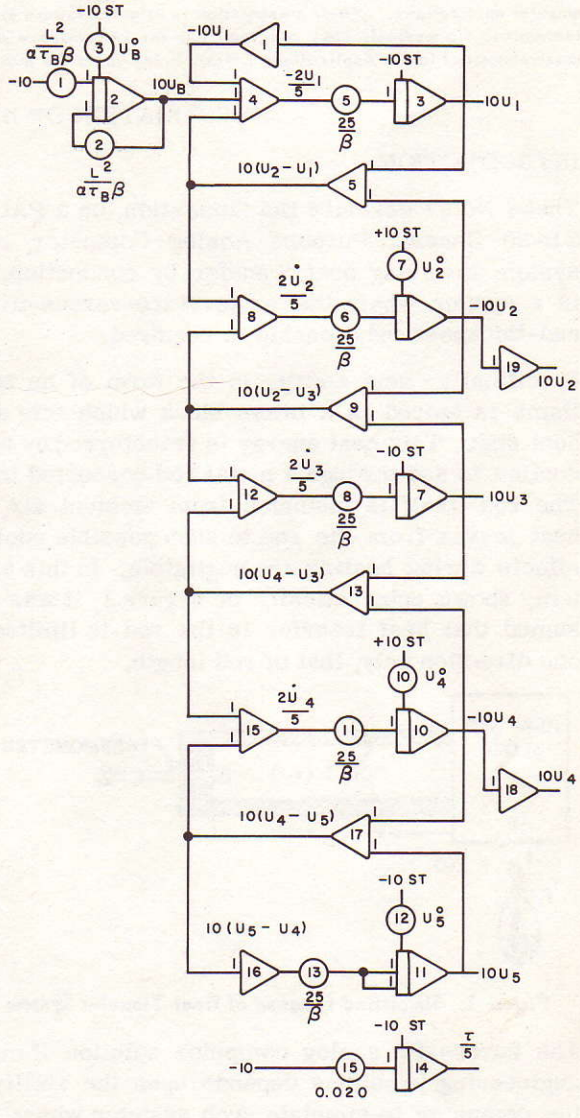
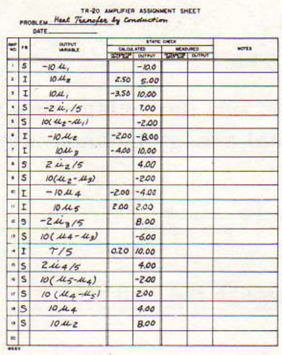


Figure 3: Computer Diagram



TR-20 AMPLIFIER ASSIGNMENT SHEET

PROBLEM Heat Transfer by Conduction

DATE

AMP NO.	FB	OUTPUT VARIABLE	STATIC CHECK: CALCULATED: INTEGRATOR INPUT SUM	CALCULATED: OUTPUT	MEASURED: INTEGRATOR INPUT SUM	MEASURED: OUTPUT	NOTES
1	S	$-10u_1$		-10.0			
2	I	$10u_B$	2.50	5.00			
3	I	$10u_1$	-3.50	10.00			
4	S	$-2\dot{u}_1/5$		7.00			
5	S	$10(u_2-u_1)$		-2.00			
6	I	$-10u_2$	-2.00	-8.00			
7	I	$10u_3$	-4.00	10.00			
8	S	$2\dot{u}_2/5$		4.00			
9	S	$10(u_2-u_3)$		-2.00			
10	I	$-10u_4$	-2.00	-4.00			
11	I	$10u_5$	2.00	2.00			
12	S	$-2\dot{u}_3/5$		8.00			
13	S	$10(u_4-u_3)$		-6.00			
14	I	$\tau/5$	0.20	10.00			
15	S	$2\dot{u}_4/5$		4.00			
16	S	$10(u_5-u_4)$		-2.00			
17	S	$10(u_4-u_5)$		2.00			
18	S	$10u_4$		4.00			
19	S	$10u_2$		8.00			
20							

Figure 5: Amplifier Assignment Sheet

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